



Matrix equations and systems of equations.

Part A: Graphing/matrix calculator allowed.

1. Solve. Check your answers.

$$\begin{cases} -x + 2y - 3z = -6 \\ 2x - y - z = -3 \\ x + 3y - 5z = -9 \end{cases}$$

(a) Write as an augmented matrix.

$$\left[ \begin{array}{ccc|c} -1 & 2 & -3 & -6 \\ 2 & -1 & -1 & -3 \\ 1 & 3 & -5 & -9 \end{array} \right]$$

(b) Solve using the reduced row echelon form (rref).

$$\boxed{x = 6/11 \quad y = 15/11 \quad z = 30/11}$$

2. You can purchase peanuts for \$2.50 per pound, almonds for \$6.00 per pound, and cashews for \$8.00 per pound. You want to make a 1,000-pound mixture that costs \$5.00 per pound.

(a) If a ratio ( $r$ ) of 3 pounds of peanuts to one pound of cashews is used, how many pounds of each type should be used? Answer to the nearest pound.

$$p = 3c$$

Let

$p$  = # pounds of peanuts  
 $a$  = # pounds of almonds  
 $c$  = # pounds of cashews.

Write the equations.

$$\begin{cases} p + a + c = 1000 \\ 2.5p + 6a + 8c = 5000 \\ p - 3c = 0 \end{cases}$$

$$\left[ \begin{array}{ccc|c} 1 & 1 & 1 & 1000 \\ 2.5 & 6 & 8 & 5000 \\ 1 & 0 & -3 & 0 \end{array} \right]$$

R<sub>2</sub> ← R<sub>2</sub> - 2.5R<sub>1</sub>

Write the answers in words, to the nearest tenth of a pound

$$\boxed{\begin{array}{l} 352.9 \text{ pounds of peanuts} \\ 529.4 \text{ pounds of almonds} \\ 117.6 \text{ pounds of cashews} \end{array}}$$

NO

Part B: Graphing calculator. Regular calculator OK.

3. Consider the matrix

$$A = \begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}$$

(a) Write the determinant. Find it.

$$\begin{vmatrix} 1 & -1 \\ -2 & 6 \end{vmatrix} = 1 \cdot 6 - (-2)(-1) \\ = 6 - 2 = \boxed{4}$$

(b) Showing the math, find the inverse.

$$A^{-1} = \frac{1}{4} \begin{bmatrix} 6 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix}$$

(c) Use the inverse of the matrix  $A$  to solve for the matrix  $X$ .

$$AX = B$$

$$\begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix} X = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

Write the equation for  $X$ .

$$X = A^{-1}B = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$$

Write  $X$ .

$$X = \begin{bmatrix} \left(\frac{3}{2}\right)(-2) + \left(\frac{1}{4}\right)(1) & \left(\frac{3}{2}\right)(3) + \left(\frac{1}{4}\right)(2) \\ \left(\frac{1}{2}\right)(-2) + \left(\frac{1}{4}\right)(1) & \left(\frac{1}{2}\right)(3) + \left(\frac{1}{4}\right)(2) \end{bmatrix}$$

$$X = \begin{bmatrix} -\frac{11}{4} & 5 \\ -\frac{3}{4} & 2 \end{bmatrix}$$

(d) Solve using the inverse matrix method.

$$\begin{cases} x - y = 5 \\ -2x + 6y = -24 \end{cases}$$

Rewrite the system of equations as a matrix equation.

$$\begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

Write the solution equation.

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 6 \end{bmatrix}^{-1} \begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

Write  $x$  and  $y$ .

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{3}{2} & \frac{1}{4} \\ \frac{1}{2} & \frac{1}{4} \end{bmatrix} \begin{bmatrix} 5 \\ -24 \end{bmatrix}$$

$$= \begin{bmatrix} \left(\frac{3}{2}\right)(5) + \left(\frac{1}{4}\right)(-24) \\ \left(\frac{1}{2}\right)(5) + \left(\frac{1}{4}\right)(-24) \end{bmatrix} \\ = \begin{bmatrix} \frac{15}{2} - 6 \\ \frac{5}{2} - 6 \end{bmatrix} = \begin{bmatrix} \frac{3}{2} \\ -\frac{7}{2} \end{bmatrix}$$

Check:

$$\begin{aligned} 3/2 - -7/2 &= 5 \\ -2 \cdot 3/2 + 6 \cdot -7/2 &= -24 \end{aligned}$$