Name KEY

Sheet 455: Review Matrices

Per/Sec. _____

Always show work. For calculator questions, show the matrices used.

Graphing calculator not allowed

1. $\begin{vmatrix} 4 & 1 \\ 3 & 2 \end{vmatrix} = 4\cdot 2 - (3\cdot 1) = \boxed{5}$

2. $\begin{vmatrix} -2 & 2 & 3 \\ 5 & 0 & 1 \\ 6 & -4 & -4 \end{vmatrix} = 0 + 2\cdot 1\cdot 6 + 5(-4)(3) - 0 - 5(2)(-4) - (-2)(-4)(1)$
$= 12 - 60 + 40 - 8 = \boxed{-16}$

3. Solve using Cramer's Method. $x + 6y = -9$, $x - 3y = 6$

$A = \begin{bmatrix} 1 & 6 \\ 1 & -3 \end{bmatrix}$. $|A| = (1)(-3) - (1)(6) = -9$

$x = \dfrac{\begin{vmatrix} -9 & 6 \\ 6 & -3 \end{vmatrix}}{-9} = \dfrac{27-36}{-9} = \dfrac{-9}{-9} = 1$

$y = \dfrac{\begin{vmatrix} 1 & -9 \\ 1 & 6 \end{vmatrix}}{-9} = \dfrac{6+9}{-9} = \dfrac{15}{-9} = \dfrac{-5}{3}$

Graphing calculator allowed $\boxed{(x,y) = (1, \frac{-5}{3})}$

4. Solve using the inverse matrix. $-3a + b = -3$, $5a - 2b = 10$

$\begin{bmatrix} -3 & 1 \\ 5 & -2 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} -3 \\ 10 \end{bmatrix}$, $A^{-1} = \dfrac{1}{|A|}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}$

$\begin{vmatrix} -3 & 1 \\ 5 & -2 \end{vmatrix} = 6 - 5 = 1$

$\begin{bmatrix} a \\ b \end{bmatrix} = \dfrac{1}{1}\begin{bmatrix} -2 & -1 \\ -5 & -3 \end{bmatrix}\begin{bmatrix} -3 \\ 10 \end{bmatrix} = \begin{bmatrix} -2(-3) - 1(10) \\ -5(-3) - 3(10) \end{bmatrix} = \begin{bmatrix} -4 \\ -15 \end{bmatrix}$

$\boxed{(a,b) = (-4, -15)}$

5. $4x + y = -7$
$x - 2z = 4$
$3y + 2z = -3$

rref of $\begin{bmatrix} 4 & 1 & 0 & -7 \\ 1 & 0 & -2 & 4 \\ 0 & 3 & 2 & -3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ -3 \end{bmatrix}$ $\boxed{(x,y,z) = (-2, 1, -3)}$

6. A local theater charges for \$2.25 for general admission and \$3.50 for box office seats. For a certain performance, 165 tickets were sold, which brought in a total of \$482.50. Find out how many of each type of ticket were sold.

$\begin{cases} 2.25g + 3.50\cdot b = 482.50 \\ g + b = 165 \end{cases}$

$b = 165 - g$ (I picked substitution method)

$2.25g + 3.5(165 - g) = 482.5$

$95 = 1.25g \rightarrow g = 76$

$\boxed{76 \text{ general}, 89 \text{ box office}}$

7. From two kinds of corn meal, a one costing \$0.65/kg and the b other \$0.85/kg, a cattle farmer wishes to make a 100 kilogram mixture costing \$0.76/kg. How many kilograms of each kind should be used?

$\begin{cases} a + b = 100 \\ 0.65a + 0.85b = 0.76\cdot 100 \end{cases}$

$\begin{bmatrix} 1 & 1 \\ 0.65 & 0.85 \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 100 \\ 76 \end{bmatrix}$

rref of $\begin{bmatrix} 1 & 1 & 100 \\ 0.65 & 0.85 & 76 \end{bmatrix}$

$\boxed{\begin{array}{l} \text{Cornmeal 1} = 45 \text{ kg at } 0.65/\text{kg} \\ \text{Cornmeal 2} = 55 \text{ kg at } \$0.85/\text{kg} \end{array}}$

8. An airline maintains three different classes of service in a Boeing 747: first, business, and economy. The configuration of the plane (number of seats in each class) is based on demand. There are 42 seats available for either first or business class, and 570 seats for either business or economy class. If, on a certain flight, there are 34 times as many economy as first class seats, what is the configuration of the plane?

$\begin{cases} f + b = 42 \\ b + e = 570 \\ 34f - e = 0 \end{cases}$

rref of $\begin{bmatrix} 1 & 1 & 0 & 42 \\ 0 & 1 & 1 & 570 \\ 34 & 0 & -1 & 0 \end{bmatrix}$

$\boxed{\begin{array}{l} \text{first class} = 16 \text{ seats} \\ \text{business class} = 26 \text{ seats} \\ \text{economy class} = 544 \text{ seats} \end{array}}$

checks =
$16 + 26 = 42$
$26 + 544 = 570$
$16 \cdot 34 = 544$

Answer

1. 5
2. −16
3. $(1, -\frac{5}{3})$
4. $(-4, -15)$
5. $(-2, 1, -3)$

76 general, 89 box
55 kg at \$0.85, 45 kg at \$0.65
16 first class, 26 business, 544 economy