Name: KEY

Sheet # 532: Logarithms and Graphs

1. Rewrite the equation in exponential form:
$\log_2 64 = 6$.

$2^6 = 64$

2. Evaluate the expressions:

(a) $\log(10{,}000) = 4$ [because $10^4 = 10{,}000$]

(b) $\log_3(27) = 3$ [because $3^3 = 27$]

(c) $\ln e^5 = 5$

3. Simplify the expressions:

(a) $9^{\log_9 x} = x$

(b) $e^{\ln(\pi x)} = \pi x$

4. Using calculator, evaluate to **three decimals,** rounding correctly:

(a) $\log(2) = 0.301$

(b) $\log(11) = 1.04$

(c) $\log(2) + \log(11) = 1.342$

(d) $\log(22) = 1.342$

What is going on here?

Property of Logarithms:
$\log(uv) = \log(u) + \log(v)$

5. Expand the expressions *completely*:

(a) $\log_4\left(\frac{2x}{z}\right)$

$\log_4(2) + \log_4(x) - \log_4(z)$

[where $\log_4(2) = 1/2$ because $4^{1/2} = 2$]

(b) $\ln(17x^5y^2)$

$\ln(17) + 5\ln x + 2\ln y$

(c) $\ln\left(\sqrt{e^4 x}\right) =$

$\ln\left(\sqrt{e^4}\right) + \ln(x)$

$= \ln(e^2) + \ln(x) = 2 + \ln x$

6. Condense the expressions into *one* logarithm:

(a) $\log(x^5) + \log(y)$

$\log(x^5 \cdot y)$

(b) $\ln(5) - 7\ln(x) + 2\ln(y)$ → $-\ln(x^7)$ → $\ln(y^2)$

$\ln\left(\frac{5y^2}{x^7}\right)$

(c) $\log(x^3) + \log(x^2) + \log(x)$

$\log(x^3 \cdot x^2 \cdot x) = \log(x^6)$

7. Consider $f(x) = \log_8(x+3)$.

(a) Find $f(61)$. That is, find y when $x = 61$.

$y = f(61) = \log_8(61+3) =$

$= \log_8(64) = 2$ because $8^2 = 64$.

(b) Write the equation of the asymptote.

$x = -3$ because the asy. of $y = \log_b(x-H)$ is $x = H$. In this case $H = -3$.

[Note: ↙ -3 is not a correct answer]

8. (a) Complete the table.

x	$y = \log_4 x$
1/16	−2
1/4	−1
1	0
4	1
16	2

because...

$4^{-2} = 1/4^2 = 1/16$

$4^{-1} = 1/4$

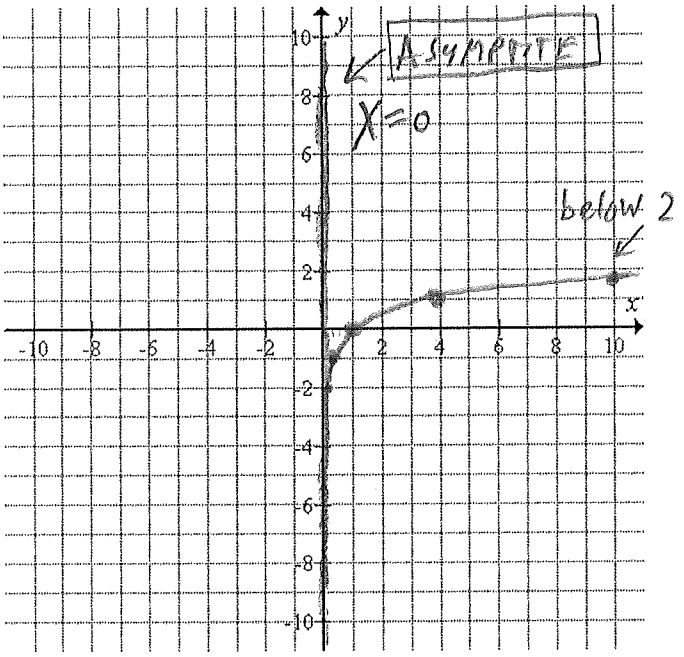
$4^0 = 1$

$4^1 = 4$

$4^2 = 16$

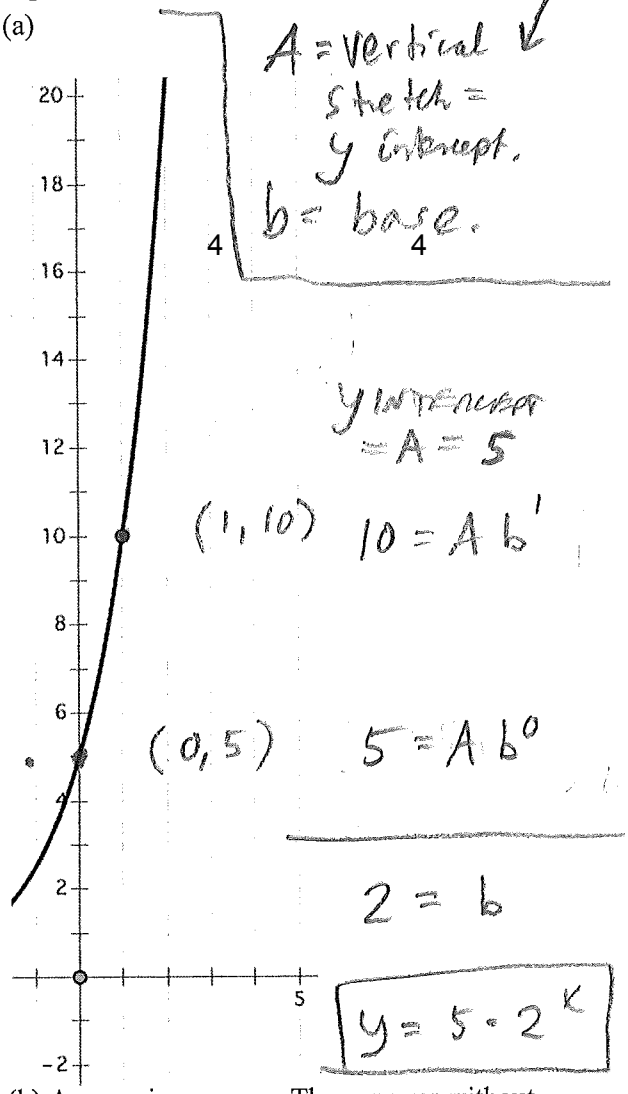
✓ (b) Graph the function y in part (a) using accurate points to show the full curve.

✓ (c) Graph the asymptote and label it "asymptote."



9. Write possible formulas for the graphs. Use exponential functions in the form $y = A \cdot b^x$.

(a)



(b) Answer in any way. Then answer without decimals and without radicals (nth roots), if you have not already done so.

↳ 96, TWO EQUATIONS TWO UNKNOWNS. HINTS:

(1) START WITH LARGEST x VALUE.

(2) DIVIDE EQUATIONS, RATHER THAN USE SUBSTITUTION (Which also works)

$y = Ab^x$

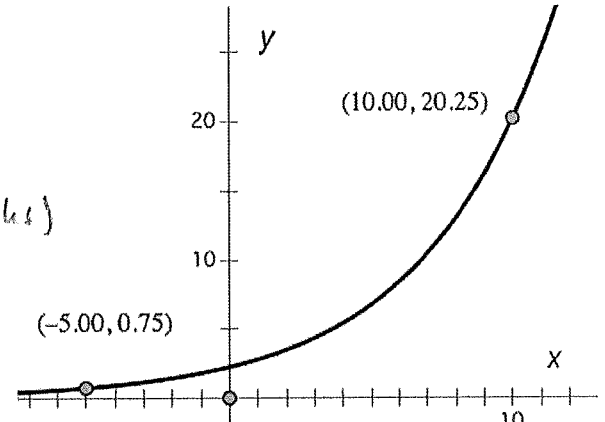
$20.25 = A b^{10}$

$0.75 = A b^{-5}$ DIVIDE

$27 = 1 \cdot b^{10-(-5)}$

$27 = b^{15}$

$\sqrt[15]{27} = b$



$b = (27)^{1/15} = (3^3)^{1/15} = 3^{(3/15)} = 3^{1/5}$

Use (10, 20.25) TO FIND A.

$20.25 = A(3^{1/5})^{10}$ $20.25 = A\,3^2$

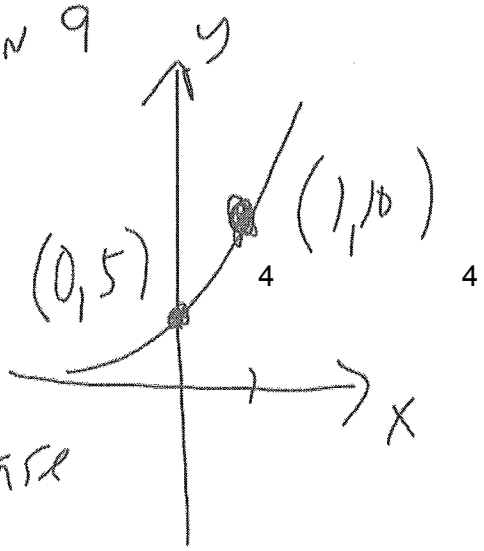
$A = 20.25/9 = 2.25$

$y = 2.25(3^{1/5})^x$

$y = 2.25\,3^{x/5}$

Sheet 533 QUESTION 9

9a)



Two points: (1,10) (0,5)

$y = A b^x$

A = y int. b = base

A = 5

What are you multiplying by when adding 1 to x?

b = 2.

ALWAYS works. USE POINTS.

PUT LARGEST x on TOP

$(1,10)$ $10 = A \cdot b^1$

$(0,5)$ $5 = A \cdot b^0$ DIVIDE

$2 = 1 \cdot b^1$

$b = 2$

$A = 5$ from $(0,5)$.

9b, See the "long" Method of 9a.

$y = Ab^x$

Use both points on graph

$20.25 = Ab^{10}$

$0.75 = Ab^{-5}$

DIVIDE

(i) Getting an answer.

$27 = 1 \cdot b^{15}$

$b = \sqrt[15]{27}$

$y = A\left(\sqrt[15]{27}\right)^x$

FIND A:

$20.25 = A\left(\sqrt[15]{27}\right)^{10}$

$A = 2.25$

$y = 2.25\left(\sqrt[15]{27}\right)^x$

(ii) The most elegant answer (IMHO)

$y = 2.25 \cdot 3^{x/5}$

4 4