Name: KEY

Sheet #558: Exponential Models 3

Score		

Possibly helpful formulas: $y = A(1+r/n)^{nt}$ $y = Ae^{kt}$ $y = A(2)^{t/T}$ $y = A(0.5)^{t/T}$

1. Solve for the unknowns to 3 significant figures. Use algebra. You may check with the calculator.

(a) $30 = 60e^{-0.05t}$

$\ln(\frac{30}{60}) / -0.05 = -0.693 / -0.05 = \boxed{13.9}$

[3 figures ↔ 1 decimal here]

(b) $120 = Ae^{0.7}$

$\frac{120}{e^{0.7}} = \boxed{59.6}$

2. The number of bacteria in a culture, y, is modeled by $y = 400e^{kt}$, where t is the time in hours.

(a) Find k if $(t, y) = (3, 600)$ is on the graph of y. Write k to ~~five~~ 6 decimals.

$600 = 400e^{k \cdot 3}$

$\ln(1.5)/3 = \boxed{0.135155}/h$

(b) Using part (a), find the number of bacteria y after $t = 4$ hours.

$686.9 = \boxed{686}$ bacteria

(c) Find the doubling time, T.

$2 = e^{kT}$ 5.12853h

$T = \ln(2)/k = \boxed{5.1285\,h}$

(d) Find the value of t for which $y = 1000$ bacteria.

$1000 = 400e^{kt}$

$t = \ln(2.5)/k =$

$= \boxed{6.7796\,h}$

6.77955h

3. The radioactive isotope Rubikskubium-299, ^{299}Rk, could have a half life of 3 attoseconds. If there were 40 femtograms of the isotope at zero attoseconds, how much of the isotope is there after the following time intervals?

(a) ~~18~~ 12 attoseconds (exactly).

$\frac{t}{T} = \frac{12}{3} = 4$ HALF LIVES. $\frac{40}{2^4} = \frac{40}{16} = \boxed{2.5\,fg}$

(b) ~~25~~ 14 attoseconds (to two decimals).

1.5749

$y = 40(0.5)^{14/3} = \boxed{1.57\,fg}$

4. A beetle population is growing exponentially. If the continuous exponential growth ***rate constant*** is 20% per day and there are 1,000 beetles now, how many beetles will there be after 14 days? Round to the nearest beetle.

$k = 0.20$ $0.2 \cdot 14$

$y = 1000e$

16,444.6

16,444

5. A student began taking the ACT at 8am ($t = 0$ hours). The student had 100% brain energy (y in percent) at that time. By 12:30pm ($t = 4.5$ hours), the student had 30% brain energy left. Use a continuous exponential decay model to find the following.

(a) Find the half life (T in hours) of brain energy. Answer to the nearest 10th of an hour.

ALT: $y = Ae^{kt}$

$y = 100(0.5)^{t/T}$

$30 = 100(0.5)^{4.5/T}$

$\ln(0.3) = \frac{4.5}{T}\ln(0.5)$

$T = 4.5\ln(0.5)/\ln(0.3) = 2.59$

2.6h

$k = \ln(0.3)/4.5$

-0.267550

$T = \ln(1.5)/k$

$= 2.5907$

(b) Predict the brain energy that was left at 10am. Answer to the nearest percent

$y = 100(0.5)^{2/2.590725}$

$= 58.56$

[Note: $100(0.5)^{2/2.6} = 58.7 \approx 59\%$] $= \boxed{59\%}$