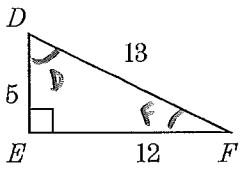
SOLUTION KEY

Name _____ Sheet #673: Trig Functions

Graphing calculator needed.

1. Find $\tan \angle D$.

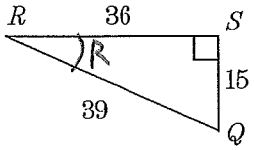


$\tan(D) = \frac{opp}{adj} = \frac{12}{5}$

2. Find $\tan \angle F$.

$\tan(F) = \frac{opp}{adj} = \frac{5}{12}$

3. Find $\sin \angle R$.



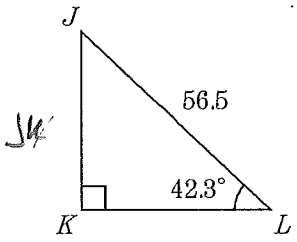
$\sin(R) = \frac{opp}{hyp} = \frac{15}{39} = \frac{5}{13}$

4. If $\sin \angle B = 0.9563$, find $m\angle B$ to the nearest degree.

$B = \sin^{-1}(0.9563) \approx 72.999^\circ$

$\approx \boxed{73.0^\circ}$ (DON'T USE 1.274 radians)

5. Find JK to the nearest tenth.



$\frac{JK}{56.5} = \sin(42.3^\circ)$

$JK \approx 56.5 \cdot 0.67301 \approx \boxed{38.0}$

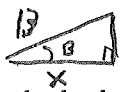
6. Find KL to the nearest tenth.

$KL = 56.5 \cdot \cos(42.3^\circ) \approx \boxed{41.8}$

7. Find $m\angle J$ to the nearest tenth.

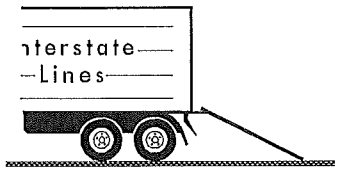
$180^\circ - 90^\circ - 42.3^\circ = \boxed{47.7^\circ}$

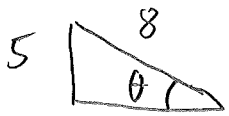
8. If $\sin \angle B = \frac{5}{13}$, find $\cos \angle B$.



$x^2 + 5^2 = 13^2 \quad x = 12 \quad \cos(B) = \frac{12}{13}$

9. One end of a ramp is raised to the back of a truck 5 feet above the ground. If the length of the ramp is 8 feet, what is the approximate measure of the angle the ramp makes with the ground? Round your answer to the nearest tenth of a degree.



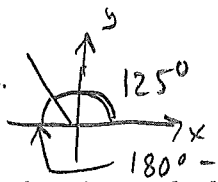


$\sin\theta = \frac{5}{8}$

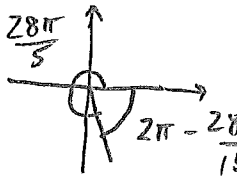
$\theta = \sin^{-1}\left(\frac{5}{8}\right) \approx \boxed{38.7^\circ}$

Name the quadrant and reference angle of the given angle.

10. a) $125°$ b) $28\pi/15$

10a) 

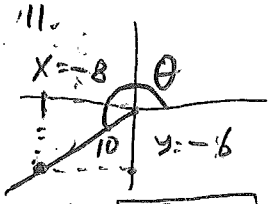
$180° - 125° = \boxed{55°\ Q\text{II}}$

10b) 

$2\pi - \frac{28\pi}{15} = \frac{2}{15}\pi$ $\boxed{\frac{2\pi}{15}\ Q\text{IV}}$

11. Find 3 functions (not 6). The terminal side of an angle α in standard position passes through the point $(-8,-6)$. Find the six trigonometric functions of α.

$\sin\theta = y/r$. $\cos\theta = x/r$. $\tan\theta = y/x$.

11. 

$r = \sqrt{6^2+8^2} = 10$

$\sin\theta = \frac{-6}{10} = \boxed{\frac{-3}{5}}$

$\cos\theta = \frac{-8}{10} = \boxed{\frac{-4}{5}}$

$\tan\theta = \frac{-3}{-4} = \boxed{\frac{3}{4}}$

10b, ALT. METHOD USING DEGREES:

$\frac{28\pi}{15} \cdot \frac{180}{\pi} =$

$= 336°$

$\theta_{ref} = 360° - 336°$

$\theta_{ref} = 24° \frac{\pi}{180°}$

$= \boxed{\frac{2\pi}{15}}$

12. Complete the following chart. Assume $0 \le \theta < 2\pi$.

radius r	central angle θ	arc length s
7	$\frac{3\pi}{8}$	$7\left(\frac{2\pi}{6}\right) = \boxed{\frac{21\pi}{6}}$
4	$\frac{2\pi}{4} = \boxed{\pi/2}$	2π
$\frac{7\pi}{2}$	$\frac{7\pi}{10}$	$\frac{7\pi}{2}$
1	$\frac{2\pi}{3}$	$1\left(\frac{2\pi}{3}\right) = \boxed{\frac{2\pi}{3}}$
2	$\frac{7\pi}{4}/2 = \boxed{\frac{7\pi}{8}}$	$\frac{7\pi}{4}$

$\frac{7\pi}{2} \Big/ \frac{7\pi}{10} = \frac{10}{2} = \boxed{5}$

13. Through how many radians will the hour hand of a clock rotate in

a) 24 hours $2(2\pi) = \boxed{4\pi}$

b) 6 hours $\frac{1}{2}(2\pi) = \boxed{\pi}$

c) 12 hours $1(2\pi) = \boxed{2\pi}$

d) 8 hours $\frac{8}{12}(2\pi) = \frac{2}{3}(2\pi) = \boxed{\frac{4\pi}{3}}$

Find the amplitude and period of the function.

14. $y = \frac{2}{3}\sin(\theta) + 1$ $A = \boxed{\frac{2}{3}}$ $T = \boxed{2\pi}$

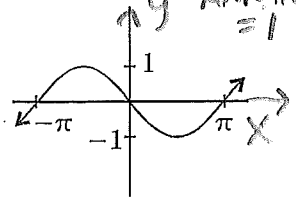
Find the phase shift and vertical shift of the function.

15. $y = 2 + \cos\left(\theta - \frac{\pi}{3}\right)$ $K = \boxed{2}$ $Sh = \boxed{\frac{\pi}{3}}$ $\left(B = 1, h = \frac{\pi}{3}\right)$

Find the amplitude and period of the function.

16. $y = \frac{1}{1}\cos\left(\frac{1}{2}x\right) + 8$ $A = \boxed{1}$ $T = \frac{2\pi}{B} = \frac{2\pi}{1/2} = \boxed{4\pi}$

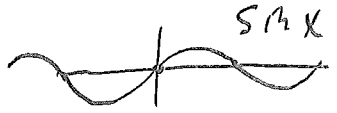
Write the equation of the graph.

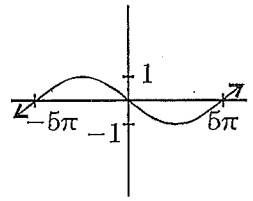
17. 

Amplitude $= 1$ $T = \pi - (-\pi) = 2\pi$ $B = 2\pi/T = 2\pi/2\pi = 1$

$y = -1.5\sin(x) + 0$

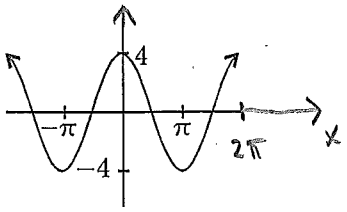
$\boxed{y = -\sin(x)}$



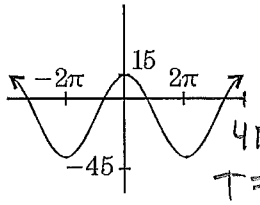
18. 

Amplitude $= 1$ $T = 10\pi$ $B = 2\pi/10\pi = 1/5$

$\boxed{y = -\sin\left(\frac{1}{5}x\right)}$

19. 

$y = 4\cos(x)$

20. 

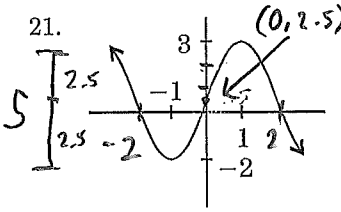
Peak-to-peak = MAX − MIN = 15 − (−45) = 60.

AMPLITUDE = 30.

VERTICAL SHIFT, $K = \frac{15 + -45}{2} = \frac{-30}{2} = -15$.

$T = 4\pi$, $B = \frac{2\pi}{4\pi} = \frac{1}{2}$

$y = 30\cos\left(\frac{x}{2}\right) - 15$

21. 

$+\sin(x)$

$T = 4$, $B = 2\pi/4 = \pi/2$.

$A = 2.5$, $K = \frac{3-2}{2} = \frac{1}{2}$

$y = 2.5\sin\left(\frac{\pi}{2}x\right) + 0.5$

22. Consider the function $y = 5 - 2\cos 3(x + \frac{\pi}{6})$. Without actually graphing the function, write an explanation of how the constants 5, −2, 3, and $\frac{\pi}{6}$ affect the graph, using the graph of $y = \cos x$ as a basis for comparison.

VERT. SHIFT $K = 5$. AMPLITUDE = VERTICAL STRETCH = 2, REFLECT ABOUT x-AXIS (−2). HORIZ. COMPRESSION = $B = 3$.

23. Note: frequency $f = 1/T$ and angular frequency $B = 2\pi/T$. The voltage E in an electrical circuit is given by $E = 5\cos 120\pi t$, where t is time measured in seconds.

HORIZONTAL SHIFT LEFT = $H = -\pi/6$

PHASE SHIFT LEFT = $BH = -\pi/2$

a) Find the amplitude and the period of the function.

AMPLITUDE = $5\frac{V}{m}$ $T = 2\pi/120\pi = 1/60$ seconds

b) What is the frequency (number of cycles completed in one second)?

$f = 1/T = 60$ cycles per second.

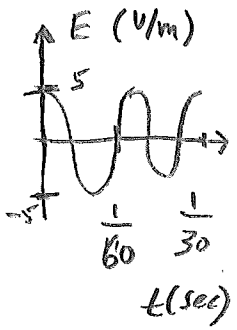
c) Find E when $t = 0, 0.03, 0.06, 0.09, 0.12$

23c)

t(sec)	E ($\frac{V}{m}$)
0	5
1.5 periods = 0.03	1.545
0.06	−4.045
0.09	−4.045
0.12	1.545
9 periods = 0.15	5

d) Graph E, for $0 \le t \le \frac{1}{30}$. = 2 periods.

23d)



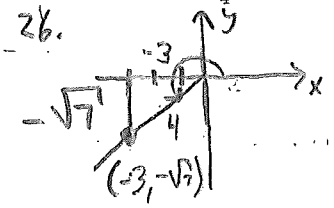
Solve.

24. $2\cos\theta + 1 = 0$

25. $\sin\theta - \frac{\sqrt{3}}{2} = 0$

26. If $\cos\theta = -\frac{3}{4}$ and θ lies in quadrant III, find $\sin\theta$.

26.



$\cos\theta = \frac{(-3)}{4}$

take $x = -3$, for example...

$r = 4$

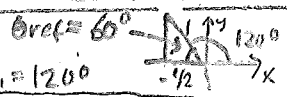
$y = \sqrt{r^2 - x^2} = \sqrt{16 - 9} = \sqrt{7}$

$\sin\theta = -\sqrt{7}/4$

24. $2\cos\theta = -1$

$\cos\theta = -1/2$

$\theta_{ref} = 60°$



30°-60°-90° Δ

$\theta_1 = 120°$

$\theta_1 = \cos^{-1}(-1/2) = 2\pi/3 \approx 2.0944$.

$\theta_1 + \theta_2 = 2\pi$. $\theta_2 = 2\pi - 2\pi/3 = \frac{6-2}{3}\pi = \frac{4\pi}{3} \approx 4.1888$.

All other solutions can be found by adding multiples of $T = 2\pi$.

25. $\sin\theta = \sqrt{3}/2$ $\theta_1 = 60°$

$\theta_1 = \sin^{-1}(\sqrt{3}/2) = \pi/3 \approx 1.0472$

$\theta_2 = \pi - \theta_1 = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3} \approx 2.0944$.

All other solutions can be found by adding multiples of $T = 2\pi$

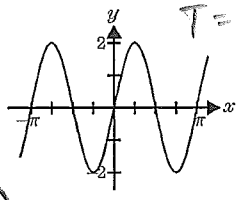
NOTE:

$\frac{\pi}{2} - \theta_1 = \theta_2 - \frac{\pi}{2}$

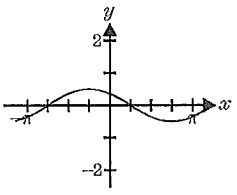
$\pi = \theta_1 + \theta_2 \rightarrow \theta_2 = \pi - \theta_1$

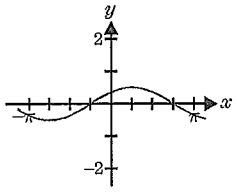
T = 2π. A = 2. SHIFT RIGHT +π/4

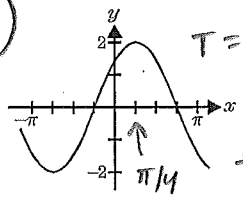
27. Which of the following is the graph of $y = 2\cos(x - \frac{\pi}{4})$?

a) 

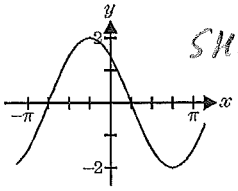
T = π

b) 

c) 

d) 

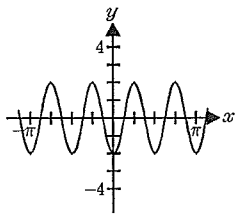
T = 2π. SHIFT RIGHT +π/4

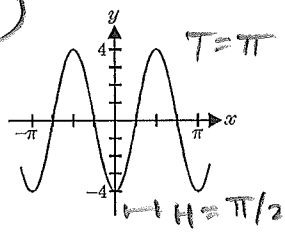
e) 

SHIFT LEFT

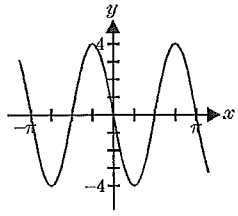
28. Which of the following is the graph of $f(x) = 4\cos(2x - \pi)$?

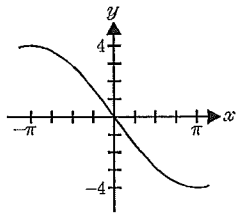
A = 4 T = π B = 2 BH = π (Arm formula) H = π/2

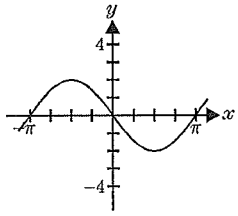
a) 

b) 

T = π
H = π/2

c) 

d) 

e) 

29. This graph illustrates a sine function for one complete cycle. Which of the following is the equation of this graph?

A = 2 H = π/8

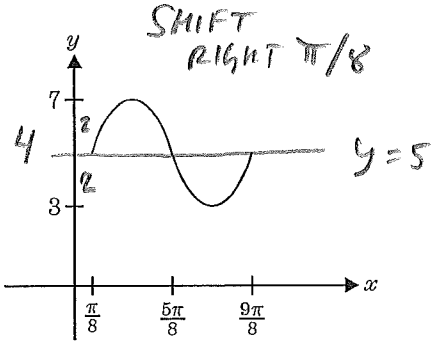
a) $y = 2\sin(2x - \frac{\pi}{4}) + 5$

b) $y = 2\sin(2x + \frac{\pi}{4}) + 5$

c) $y = 3\sin(x - \frac{\pi}{8}) + 5$

d) $y = 3\sin(x + \frac{\pi}{8}) + 5$

e) $y = 2\sin(x - \frac{\pi}{4}) + 5$



SHIFT RIGHT π/8

y = 5

30. Solve for x algebraically. Work in radians. Give any one solution. Answer with 3 decimals. Show your work with the appropriate inverse trig function. You may check your work with the calulator.

a) $5 = 3\cos(x) + 4$

<See answer key>

b) $9 = 5\sin(4\pi x) + 12$

<See answer key>